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$N = 1$ SUPERFIELDS AND $N = 2$ HARMONIC SUPERFIELDS IN FOUR DIMENSIONS AS SECOND QUANTIZED SUPERPARTICLES

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Supersymmetric $N = 1,2$ point particle actions in four dimensions, substantially differing from the Siegel's one, are proposed as constrained dynamical systems. It is shown that upon second quantization they yield the superfield theories of the $N = 1$ chiral- and vector supermultiplets as well as the $N = 2$ matter and super-Maxwell multiplets in the harmonic superspace formalism, In particular, there arises an alternative understanding of the concepts of harmonic analyticity and harmonic charges of $N = 2$ superfields in terms of classical superparticle mechanics.

1. Introduction and Motivation

Quantum theories of the ordinary scalar field and the bosonic string field indicate a general pattern of transition via the process of first- and second quantization from classical relativistic dynamical systems with constraints, describing single objects (point particles or strings), to the corresponding gauge-invariant quantum field theories, whose quanta are these objects (see, e.g. Ref. 1). In particular, a very important property is the connection between the gauge symmetries of the classical single-object actions and the gauge symmetries of the corresponding field actions which is elucidated by means of the first-quantized BRST charge.[2,3]

In the context of superfield- and superstring field theories there has been much interest in discussing Siegel's superparticle[4–6] both due to the fact that it represents a "zero-mode" approximation to the covariant Green-Schwarz superstring,[7] as well as because of the presence of a local fermionic symmetry playing a fundamental role both in the superparticle- and superstring case.

Siegel's superparticle, whose classical action reads:[4]

$$S = \int d\tau [p_\mu \partial_\tau x^\mu - p_\theta \partial_\tau \theta - \lambda p^2 - \psi \not{p} d], \tag{1}$$

(where λ and ψ are Lagrange multipliers and the Grassmann spinor θ is Majorana or Majorana-Weyl), describes upon second quantization physical supermultiplets only on-shell[4] (cf. Eq. (2) below). Recently the action (1) as well its $N = 2$ generalization were reconsidered[8] taking properly into account the functional dependence, i.e. the reducibility in the sense of Ref. 9, of the two first class constraints:

$$p^2 = 0, \qquad \not{p}d \equiv \not{p}(ip_\theta - \not{p}\theta) = 0,$$
$$\{\not{p}d, \not{p}d\}_{\text{P.B.}} = (2i\not{p})p^2, \tag{2}$$

where $\{\ ,\ \}_{\text{P.B.}}$ denotes graded Poisson bracket.[10] It was found that consistent Lorentz-covariant canonical first- and second quantization of (1) off-shell is only possible after:

(i) the introduction of bosonic Lorentz-spinor "harmonic" variables (in addition to the usual superspace canonical coordinates (x, θ)), serving to covariantly separate the half independent constraints from $\not{p}d = 0$ in (2);

(ii) an appropriate modification of (1) or its $N = 2$ version to ensure that these harmonics are pure gauge degrees of freedom. Upon second quantization this $N = 2$ generalization of (1) with Lorentz-spinor harmonics yields a covariant description of $D = 10$ type IIb supergravity[11] in terms of unconstrained superfields along the lines of the covariant BRST second quantization scheme.[3] Also, the first-quantized BRST charge of the modified superparticle contains higher order multighost terms.[8]

There exists another modification[6] of the action (1) in $D = 10$ where additional Lorentz-vector harmonic variables are added and it describes after second quantization certain type of $D = 10$ covariant light-cone harmonic superfield.

Our aim in the present note is to discuss the underlying superparticle actions in the ordinary $D = 4$ case which upon second quantization yield off-shell the free massless superfield theories of the $N = 1$ chiral- and vector supermultiplets as well as the $N = 2$ matter- and super-Maxwell multiplets in the harmonic superspace framework.[12,13] Here we propose the relevant $D = 4$ classical superparticle actions with $N = 1$, 2 supersymmetry which possess besides reparametrization invariance various types of local fermionic and/or local bosonic symmetries. These actions substantially differ from (1). While in the $N = 1$ case the meaning of the local symmetries at the classical level remains obscure, at the quantum level they constrain the superfields to describe irreducible supermultiplets. In the $N = 2$ case the local symmetries provide an alternative simple way to understand from the point of view of classical superparticle mechanics the origin of the concepts of harmonic analyticity and harmonic charges of the $N = 2$ unconstrained harmonic superfields.[12]

As a final introductory remark let us recall the general ideology for covariant second quantization of constrained dynamical systems, starting from the covariant first-quantized Dirac constraint equations.[1] One of the constraint equations containing $\hat{p}^2 = -\Box$ is interpreted as field equation of motion and the rest are viewed as gauge-fixing conditions for certain gauge invariance(s) (that has(have) to be discovered) of the corresponding field theory. A necessary condition for this scheme to provide an off-shell, i.e. action-principle, description of the second-quantized theory is that the subset of first-quantized constraints which does not contain the "dynamical" constraint (the constraint involving $\hat{p}^2 = -\Box$) must form a closed subalgebra under commutation. Such constraints are also called "kinematical". Clearly, the Siegel's fermionic constraints are not kinematical as seen from (2), unlike the constraints presented below.

In our notation we follow Refs. 14, 12.

2. $N = 1$ Superparticles in $D = 4$

Let us consider the following $N = 1$ superparticle actions:

$$S_{V,ch} = \int d\tau [p_\mu \partial_\tau x^\mu + p_\theta^\alpha \partial_\tau \theta_\alpha + \bar{p}_{\theta\dot{\alpha}} \partial_\tau \bar{\theta}^{\dot{\alpha}} - H_{V,ch}], \tag{3}$$

$$H_V = \lambda p^2 + \tfrac{1}{2}\chi(d^\alpha d_\alpha + \bar{d}_{\dot{\alpha}} \bar{d}^{\dot{\alpha}}), \tag{4.a}$$

$$H_{ch} = \lambda p^2 + \psi^\alpha(-\tfrac{1}{4}\bar{d}^2 d_\alpha) + \bar{\psi}_{\dot{\alpha}}(-\tfrac{1}{4}d^2 \bar{d}^{\dot{\alpha}}), \tag{4.b}$$

where

$$d_\alpha \equiv i p_{\theta\alpha} - \not{p}_{\alpha\dot{\beta}} \bar{\theta}^{\dot{\beta}}, \qquad \bar{d}_{\dot{\alpha}} \equiv i \bar{p}_{\theta\dot{\alpha}} + \not{p}_{\beta\dot{\alpha}} \theta^\beta. \tag{5}$$

Clearly, d_α, $\bar{d}_{\dot{\alpha}}$ are the classical analogues of the $N = 1$ supercovariant derivatives:

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \not{\partial}_{\alpha\dot{\beta}} \bar{\theta}^{\dot{\beta}}, \qquad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i \not{\partial}_{\beta\dot{\alpha}} \theta^\beta.$$

The graded Poisson bracket relations[10] among the canonical variables read:

$$\{p_\mu, x^\nu\}_{\text{P.B.}} = -\delta_\mu{}^\nu, \qquad \{p_\theta{}^\alpha, \theta_\beta\}_{\text{P.B.}} = \delta^\alpha{}_\beta, \qquad \{\bar{p}_{\theta\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\}_{\text{P.B.}} = \delta_{\dot{\alpha}}^{\dot{\beta}} \tag{6}$$

All objects (3)–(6) are invariant under global $N = 1$ supersymmetry transformations:

$$\delta\theta_\alpha = \varepsilon_\alpha, \qquad \delta\bar{\theta}_{\dot{\alpha}} = \bar{\varepsilon}_{\dot{\alpha}}, \qquad \delta x^\mu = -i[\bar{\varepsilon}\tilde{\sigma}^\mu \theta + \varepsilon \sigma^\mu \bar{\theta}],$$

$$\delta p_\theta{}^\alpha = i \not{p}^{\alpha\dot{\beta}} \bar{\varepsilon}_{\dot{\beta}}, \qquad \delta \bar{p}_\theta{}^{\dot{\alpha}} = -i \not{p}^{\beta\dot{\alpha}} \varepsilon_\beta, \qquad \delta p_\mu = 0.$$

Both hamiltonians $H_{V,ch}$ (4.a, b) are chosen as sums of first-class constraints with Lagrange multipliers λ, χ, ψ including the usual bosonic point-particle constraint $p^2 = 0$ corresponding to reparametrization invariance. S_V possesses local bosonic symmetry with parameter $\beta(\tau)$ generated by the nilpotent constraint $\frac{1}{2}(d^2 + \bar{d}^2)$:

$$\begin{aligned} \delta\theta_\alpha &= -i\beta d_\alpha, \qquad \delta p_\theta{}^\alpha = -\beta \not{p}^{\alpha\dot{\alpha}} \bar{d}_{\dot{\alpha}}, \qquad \delta x^\mu = -\beta[\bar{\theta}\tilde{\sigma}^\mu d + \theta \sigma^\mu \bar{d}], \\ \delta\bar{\theta}_{\dot{\alpha}} &= -i\beta \bar{d}_{\dot{\alpha}}, \qquad \delta \bar{p}_\theta{}^{\dot{\alpha}} = \beta \not{p}^{\alpha\dot{\alpha}} d_\alpha, \qquad \delta p_\mu = 0. \end{aligned} \tag{7}$$

S_{ch} possesses local fermionic symmetry with a Majorana spinor parameter $\kappa(\tau) = (\kappa_\alpha(\tau), \bar{\kappa}^{\dot{\alpha}}(\tau))$ generated by the fermionic constraint $(-\frac{1}{4}\bar{d}^2 d_\alpha, -\frac{1}{4}d^2 \bar{d}^{\dot{\alpha}})$:

$$\delta\theta_\alpha = -\frac{i}{4}\bar{d}^2 \kappa_\alpha - \frac{i}{2}(\bar{\kappa}\bar{d})\, d\alpha, \qquad \delta p_\theta{}^\alpha = -\tfrac{1}{2}(\kappa d) \not{p}^{\alpha\dot{\alpha}} \bar{d}_{\dot{\alpha}} - \tfrac{1}{4} \not{p}^{\alpha\dot{\alpha}} d^2 \bar{\kappa}_{\dot{\alpha}},$$

$$\delta\bar{\theta}_{\dot{\alpha}} = \frac{i}{4} d^2 \bar{\kappa}_{\dot{\alpha}} + \frac{i}{2}(\kappa d)\bar{d}_{\dot{\alpha}}, \qquad \delta \bar{p}_\theta{}^{\dot{\alpha}} = -\tfrac{1}{2}(\bar{\kappa}\bar{d}) p\!\!\!/^{\alpha\dot{\alpha}} d_\alpha - \tfrac{1}{4} p\!\!\!/^{\alpha\dot{\alpha}} \bar{d}^2 \kappa_\alpha,$$

$$\delta x^\mu = \tfrac{1}{2}[(\kappa d)(\theta\sigma^\mu \bar{d}) + (\bar{\kappa}\bar{d})(\bar{\theta}\bar{\sigma}^\mu d)] + \tfrac{1}{4}[\bar{d}^2(\bar{\theta}\sigma^\mu\kappa) + d^2(\theta\sigma^\mu\bar{\kappa})], \qquad \delta p_\mu = 0. \quad (8)$$

From Eqs. (7) and (8) one immediately sees that as long as the parameters of the above local symmetries appear multiplied by nilpotent functions of the canonical variables one cannot use (7) and (8) to gauge away (part of) θ_α, $\bar{\theta}_{\dot{\alpha}}$.

Although, classically, the meaning of the constraints $\frac{1}{2}(d^2 + \bar{d}^2)$ and $(-\frac{1}{4}\bar{d}^2 d_\alpha, -\frac{1}{4}d^2\bar{d}_{\dot{\alpha}})$ in $H_{V,ch}$ (4.a, b) remains unclear, it becomes very simple and transparent after quantization. Indeed, according to the general theory,[15] covariant first quantization of (3), (4.a, b) is accomplished by imposing the following constraint equations on the superfield wave function which we take as real scalar superfield $V(x, \theta, \bar{\theta})$:

$$\Box V = 0, \qquad \tfrac{1}{2}(D^2 + \bar{D}^2)V = 0 \qquad (9)$$

for S_V, and:

$$\Box V = 0, \qquad W_\alpha \equiv -\tfrac{1}{4}\bar{D}^2 D_\alpha V = 0, \qquad \bar{W}_{\dot{\alpha}} \equiv -\tfrac{1}{4}D^2\bar{D}_{\dot{\alpha}} V = 0 \qquad (10)$$

for S_{ch}, respectively.

Now, applying the general scheme for covariant second quantization[1] (as mentioned in Sec. 1) one easily infers that Eqs. (9) are precisely the equation of motion and the gauge-fixing condition for a free Maxwell superfield V describing the vector supermultiplet whose gauge-fixed action reads:[14]

$$S_{\text{gauge inv.}} + S_{\text{g.f.}} = \frac{1}{4}\int d^4x\, d^2\theta\, W^\alpha W_\alpha + \text{h.c.} - (32\alpha)^{-1}\int d^4x\, d^4\theta [(D^2 + \bar{D}^2)V]^2$$

$$= -\frac{1}{2}\int d^4x\, d^4\theta\, V\left[\Box + \left(\frac{1}{\alpha} - 1\right)\frac{1}{16}(D^2\bar{D}^2 + \bar{D}^2 D^2)\right]V,$$

and provided $\alpha = 1$ is chosen.

Likewise, we find that Eqs. (10) are precisely the equation of motion and the gauge-fixing conditions in the gauge-invariant description of the chiral superfield (see e.g. Ref. 16):

$$\Phi = \tfrac{1}{4}\bar{D}^2 V, \qquad \bar{\Phi} = \tfrac{1}{4}D^2 V. \qquad (11)$$

Indeed, according to the identity

$$\Box = \tfrac{1}{16}(D^2\bar{D}^2 + \bar{D}^2 D^2) - \tfrac{1}{8}D^\alpha \bar{D}^2 D_\alpha,$$

one finds that Eqs. (10) are equivalent to:

$$D^2\bar{D}^2 V = \bar{D}^2 D^2 V = 0 .$$

So for the "projected" chiral and antichiral superfields (11) we have

$$D^2\Phi = 0, \qquad \bar{D}^2\bar{\Phi} = 0,$$

which are precisely the equations of motion of the free chiral supermultiplet. The corresponding gauge-invariant and gauge-fixing actions for V are:

$$S_{\text{gauge inv.}} = \int d^4x\, d^4\theta\, \bar{\Phi}\Phi = -\frac{1}{32}\int d^4x\, d^4\theta\, V[D^2\bar{D}^2 + \bar{D}^2D^2]V, \qquad (12)$$

with gauge invariance $V \to V + D^\alpha\omega_\alpha + \bar{D}_{\dot\alpha}\bar{\omega}^{\dot\alpha}$ (ω_α, $\bar{\omega}_{\dot\alpha}$ being chiral (antichiral) spinor superfields);

$$S_{\text{g.f.}} = \frac{1}{4\alpha}\int d^4x\, d^2\theta\, W^2 + \text{h.c.};$$

$$S_{\text{gauge inv.}} + S_{\text{g.f.}} = -\frac{1}{2}\int d^4x\, d^4\theta\, V\left[\Box + \frac{1}{8}\left(1 - \frac{1}{\alpha}\right)D^\alpha\bar{D}^2D_\alpha\right]V,$$

and once again the choice $\alpha = 1$ is made to get Eqs. (10).

We thus conclude that from the superparticle point of view the gauge-invariant description (12) of the $D = 4$ chiral superfield (11) is more natural. In particular, the chirality constraint $\bar{D}_{\dot\alpha}\Phi = 0$ cannot be interpreted as first-quantized version of a classical first-class constraint $\bar{d}_{\dot\alpha} = 0$, since the latter would violate the reality of the classical action.

Let us also note the curious fact that the $N = 1$ chiral superfield theory may be recovered (after second quantization) from a nonrelativistic supersymmetric point particle system, the Lorentz invariance thus arising as dynamical symmetry.[17]

3. $N = 2$ Superparticles in $D = 4$

Unlike the $N = 1$ case, the $N = 2$ version of Siegel's action (1):

$$S = \int d\tau[p_\mu\partial_\tau x^\mu + p_\theta^{\alpha i}\partial_\tau\theta_{\alpha i} + \bar{p}_{\theta\dot\alpha i}\partial_\tau\bar{\theta}^{\dot\alpha i} - \lambda p^2 + \psi^{\alpha i}\not{p}_{\alpha\dot\beta}\bar{d}_i^{\dot\beta} + \bar{\psi}_{\dot\alpha}{}^i\not{p}^{\beta\dot\alpha}d_{\beta i}],$$

$$(d_\alpha^j = ip_{\theta\alpha}^j - \not{p}_{\alpha\dot\beta}\bar{\theta}^{\dot\beta j}, \quad \bar{d}_{\dot\alpha}^j = i\bar{p}_{\theta\dot\alpha}^j + \not{p}_{\beta\dot\alpha}\theta^{\beta j}), \qquad (1')$$

can serve in a sense as the starting point to construct the correct $N = 2$ superparticle actions leading to off-shell second-quantized $N = 2$ field theories.

As already stressed in Ref. 8 and in Sec. 1 above, one has to separate the half functionally independent (with respect to $p^2 = 0$) constraints from the constraints $\not{p}d_i = (\not{p}_{\alpha\dot{\beta}}\bar{d}_i^{\dot{\beta}}, \not{p}^{\dot{\alpha}\beta}d_{\beta i}) = 0$ in (1′). Here we insist on preserving simultaneously both Lorentz invariance and $N = 2$ supersymmetry with manifest SU(2) automorphism group. This can be done only at the price of introducing additional bosonic degrees of freedom. In the present case the most natural candidates for such additional degrees of freedom are the spherical harmonics $u_i^{\pm}$:[12]

$$u^{+i}u_i^- = 1, \qquad u^{\pm i}u_i^{\pm} = 0, \qquad \overline{(u^{+i})} = u_i^-, \qquad u_j^{\pm} \sim e^{\pm i\alpha}u_j^{\pm}, \tag{13}$$

The first three relations (13) define a parametrization of SU(2) and the last equivalence relation (modulo U(1) phase transformations) in (13) specifies that in fact $u_i^{\pm}$ parametrize SU(2)/U(1) $\cong S^2$.[a] With the help of $u_i^{\pm}$ (13) a Lorentz-covariant and $N = 2$ supersymmetric functionally independent (with respect of the constraint $p^2 = 0$) subset of the Siegel's fermionic constraints is taken in the form:

$$\not{p}u_i^+ d^i = (\not{p}_{\alpha\dot{\beta}}u_i^+\bar{d}^{\dot{\beta}i}, \not{p}^{\beta\dot{\alpha}}u_i^+ d_\beta^i). \tag{14}$$

Furthermore, we can drop the factors $\not{p}$ in (14) since the resulting constraints:

$$d^+ = u_i^+ d^i = (u_i^+ d_\alpha^i, u_i^+ \bar{d}^{\dot{\alpha}i}) \equiv (d_\alpha^+, \bar{d}^{+\dot{\alpha}})$$

continue to form a set of first-class constraints:[b]

$$\{d_\alpha^+, \bar{d}_{\dot{\beta}}^+\}_{\text{P.B.}} = 0, \qquad \{\overset{(-)}{d}{}^+_{\alpha(\dot{\alpha})}, p^2\}_{\text{P.B.}} = 0,$$

and, more importantly, the latter become already "kinematical" constraints.

Thus, we propose the following $N = 2$ superparticle actions in $D = 4$:

$$S_{q,l} = \int d\tau[p_\mu\partial_\tau x^\mu + p_\theta^{\alpha i}\partial_\tau\theta_{\alpha i} + \bar{p}_{\theta\dot{\alpha}i}\partial_\tau\bar{\theta}^{\dot{\alpha}i} + p_u^{-i}\partial_\tau u_i^+ - p_u^{+i}\partial_\tau u_i^- - H_{q,l}], \tag{15}$$

$$H = \lambda p^2 + \psi^{-\alpha}d_\alpha^+ + \bar{\psi}_{\dot{\alpha}}^-\bar{d}^{+\dot{\alpha}} + \rho(d^{+-} - q) + \sigma^{(-2l)}(d^{++})^l, \tag{16}$$

with the additional second-class constraints:

[a] Here we use the same SU(2) and U(1) notations and conventions for the harmonics as in Refs. 12, 13:

$$X^{\pm} \equiv u_i^{\pm}X^i, \qquad X^{(\pm n)} \equiv X^{\pm\pm\cdots\pm}\ (n \text{ indices } \pm), \qquad X_{[ij]} \equiv X_{ij} - X_{ji}, \qquad X_{(ij)} \equiv X_{ij} + X_{ji}.$$

[b] If we take constraints $d = 0$ instead of $\not{p}d = 0$ in the action (1) then, because of the relation $\{d_\alpha, \bar{d}_{\dot{\alpha}}\} = -2i\not{p}_{\alpha\dot{\alpha}}$, the constraints $d = 0$ form a set of mixed second- and first-class constraints due to the non-invertibility of the matrix $\not{p}$ on the constraint surface $p^2 = 0$. This is precisely the case of the Brink-Schwarz superparticle action (18) which has less physical degrees of freedom than (1).

$$\Phi_{ij} \equiv u^+_{[i} u^-_{j]} - \varepsilon_{ij} = 0, \qquad \chi_{ij} \equiv u^-_{[i} p^+_{uj]} - u^+_{[i} p^-_{uj]} = 0. \tag{17}$$

The basic graded Poisson bracket relations read:

$$\{p_\theta{}^{\alpha i}, \theta_{\beta j}\}_{\text{P.B.}} = \delta^\alpha{}_\beta \delta^i{}_j, \qquad \{\bar{p}_{\theta\dot{\alpha}i}, \bar{\theta}^{\dot{\beta}j}\}_{\text{P.B.}} = \delta_{\dot{\alpha}}{}^{\dot{\beta}} \delta_i{}^j, \qquad \{p_u^{\mp i}, u_j^\pm\}_{\text{P.B.}} = \mp \delta^i{}_j. \tag{18}$$

All Eqs. (15)–(18) are invariant under global $N = 2$ supersymmetry transformations:

$$\delta\theta^i_\alpha = \varepsilon^i_\alpha, \qquad \delta p^\alpha_{\theta j} = i\not{p}^{\alpha\dot{\beta}} \bar{\varepsilon}_{\dot{\beta}j}, \qquad \delta\bar{\theta}_{\dot{\alpha}i} = \bar{\varepsilon}_{\dot{\alpha}i}, \qquad \delta\bar{p}^{\dot{\alpha}}_{\theta j} = -i\not{p}^{\beta\dot{\alpha}} \varepsilon_{\beta j},$$

$$\delta x^\mu = -i[\bar{\varepsilon}^j \tilde{\sigma}^\mu \theta_j + \varepsilon^j \sigma^\mu \bar{\theta}_j], \qquad \delta p_\mu = 0, \qquad \delta u^\pm_i = 0.$$

The hamiltonian (16) once again is a sum of first class constraints with Lagrange multipliers λ, ψ^-, ρ, $\sigma^{(-2l)}$ and where:

$$d^{+-} \equiv u^{+i} p^-_{ui} + u^{-i} p^+_{ui}, \tag{19}$$

$$d^{++} \equiv -u^{+i} p^+_{ui}. \tag{20}$$

The term ρd^{+-} in (16) is necessary to maintain the local U(1) gauge-invariance of (15) which accounts for the last equivalence relation in (13), ρ being the corresponding U(1) gauge field:

$$\rho \to \rho - i\partial_\tau \alpha, \qquad p_u^{\pm j} \to e^{\pm i\alpha} p_u^{\pm j}, \qquad u^{\pm j} \to e^{\pm i\alpha} u^{\pm j}. \tag{21}$$

The constraint d^{+-} (19) in (16) is shifted by a constant q which clearly does not spoil neither the first-class property of d^{+-} nor the U(1) gauge-invariance of $S_{q,l}$ (15) under (21). The presence of the constant q may be understood as a result of ambiguity in the normal ordering of the noncommutative factors in the first quantization of d^{+-} (19). Moreover, q must be an integer since the quantized version of d^{+-}:

$$D^{+-} = u^{+i} \frac{\partial}{\partial u^{+i}} - u^{-i} \frac{\partial}{\partial u^{-i}} \tag{22}$$

possesses only integer eigenvalues (cf. Ref. 12).

Finally, we have added still another constraint $(d^{++})^l = 0$ (l being a nonzero integer power) in $H_{q,l}$ (16) which maintains the first-class property of the set of all constraints in (16):

$$\{d^{+-}, \overset{(-)}{d}{}^+_{\alpha(\dot{\alpha})}\}_{\text{P.B.}} = \overset{(-)}{d}{}^+_{\alpha(\dot{\alpha})}, \qquad \{d^{+-}, (d^{++})^l\}_{\text{P.B.}} = 2l(d^{++})^l, \qquad \text{rest} = 0. \tag{23}$$

An important property of the second-class constraints Φ_{ij}, χ_{ij} (17) which form a conjugate pair:

$$\{\Phi_{ij}, \chi^{kl}\}_{\text{P.B.}}|_{\Phi,\chi=0} = \varepsilon_{ij}\varepsilon^{kl}$$

is that they commute with all first-class constraints in $H_{q,l}$ (16) except for:

$$\{\chi_{ij}, \overset{(-)}{d}{}^{+}_{\alpha(\dot\alpha)}\}_{\text{P.B.}} = \varepsilon_{ij}\, \overset{(-)}{d}{}^{+}_{\alpha(\dot\alpha)}\,.$$

Thus, the algebra of the first-class constraints (23) remains unchanged after going on to the Dirac bracket relations.

Let us note that the objects (15)–(17) are real with respect to the combined involution $\overset{*}{\bar{}}$ introduced in Ref. 12:

$$\overset{*}{\overline{(u_i^{\pm})}} = u^{\pm i}, \qquad \overset{*}{\overline{(p_u^{\pm i})}} = -p_{ui}^{\pm}, \qquad \overset{*}{\bar{\rho}} = \rho,$$

which combines ordinary complex conjugation with flipping of the U(1) charges.

Classically, the local fermionic symmetry generated by $d^{+} = (d_{\alpha}^{\pm}, \bar{d}^{\,+\dot\alpha})$ allows one to gauge away half θ^{-} of the θ^{i}'s ($\theta^{\pm} = u_i^{\pm}\theta^{i}$)

$$\delta\,\overset{(-)}{\theta}{}^{-}_{\alpha(\dot\alpha)} = \{\kappa^{-\beta}d_{\beta}^{+} + \bar\kappa_{\dot\beta}^{-}\bar d^{\,+\dot\beta}, \overset{(-)}{\theta}{}^{-}_{\alpha(\dot\alpha)}\}_{\text{P.B.}} = \pm i\,\overset{(-)}{\kappa}{}^{-}_{\alpha(\dot\alpha)}$$

whereas the harmonic constraints d^{+-}, $(d^{++})^{l}$ can be used to gauge away only part of $u_i^{\pm}$. The meaning of these constraints becomes much more transparent after quantization.

4. $N = 2$ Harmonic Superfields from $N = 2$ Superparticles

The covariant first-quantized Dirac constraint equations or, alternatively, the second-quantized field equations of motion and the field theory gauge-fixing conditions corresponding to the classical $N = 2$ superparticle (15)–(17) read;

$$\Box V = 0, \tag{24.a}$$

$$\overset{(-)}{D}{}^{+}_{\alpha(\dot\alpha)} V = 0, \tag{24.b}$$

$$(D^{+-} - q)V = 0, \tag{24.c}$$

$$(D^{++})^{l} V = 0, \tag{24.d}$$

where $V = V(x, \theta^{j}, u)$ is a general $N = 2$ harmonic superfield and

$$D_{\alpha}^{+} = u_j^{+}\left(\frac{\partial}{\partial\theta_j^{\alpha}} + i\partial_{\alpha\dot\beta}\bar\theta^{\dot\beta j}\right), \qquad \bar D_{\dot\alpha}^{+} = u_j^{+}\left(-\frac{\partial}{\partial\bar\theta_j^{\dot\alpha}} - i\partial_{\beta\dot\alpha}\theta^{\beta j}\right);$$

$$D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}},\ D^{+-} \text{ as in (22).}$$

Now, Eq. (24.b) is immediately recognized as the condition for harmonic analyticity.[12] Next, Eq. (24.c) clearly represents the condition that V carries a definite U(1) charge q. Both Eqs. (23.b, c) were explicitly solved in Refs. 12 in terms of charged analytic harmonic superfields. In order for Eqs. (24) to describe physical $N = 2$ supermultiplets in $D = 4$ we have to specify the values of the integers q, l.

Let us first consider the case $q = 2$, $l = 1$. Then, Eqs. (24) are easily recognized as describing the free $N = 2$ super-Maxwell multiplet V^{++}. Indeed, let us recall[12,13] the gauge-invariant action for $V = V^{++}$, where V^{++} is analytic and carries U(1) charge $q = 2$, i.e. satisfies Eqs. (24.b, c) with $q = 2$:

$$S_{\text{gauge inv.}} = \frac{1}{2}\int d\zeta^{(-4)} du_1\, du_2\, V^{++}(\zeta, u_1)(D_1^+)^4(u_1^+ u_2^+)^{-2} V^{++}(\zeta, u_2), \tag{25}$$

Here the following notations are used:

$$\zeta \equiv (x_A^\mu, \theta^+, \bar\theta^+), \qquad x_A^\mu \equiv x^\mu - 2i\theta^{(i}\sigma^\mu\bar\theta^{j)}u_i^+ u_j^-, \qquad d\zeta^{(-4)} \equiv d^4x_A\, d^4\theta^+,$$

$$(D^+)^4 \equiv (D^+)^2(\bar D^+)^2, \qquad u_1^+ u_2^+ \equiv u_1^{+i} u_{2i}^+.$$

One can choose the following gauge-fixing condition coinciding with Eq. (24.d) for $l = 1$:

$$D^{++}V^{++} = 0,$$

which corresponds to the gauge invariance of (25) under $\delta V^{++} = -D^{++}\Lambda$ with Λ being an analytic superfield gauge parameter. Then the total gauge-fixed field action for V^{++} reads:[13]

$$S_{q=2,l=1} \equiv S_{\text{gauge inv.}} + S_{\text{g.f.}} = \frac{1}{2\alpha}\int d\zeta^{(-4)} du\, V^{++}\Box V^{++}$$

$$+ \frac{1}{2}\left(1 + \frac{1}{\alpha}\right)\int d\zeta^{(-4)} du_1\, du_2\, V^{++}_{(\zeta,u_1)}(D_1^+)^4(u_1^+ u_2^+)^{-2} V^{++}_{(\zeta,u_2)},$$

which, with the choice $d = -1$, leads to Eq. (24.a) as equation of motion for V^{++}.

Following exactly the same line of analysis we find that the second-quantized $N = 2$ superparticle (15)–(17) with $q = 1$, $l = 1$ represents the field theory of the $N = 2$ Fayet-Sohnius matter supermultiplet[19] described in terms of analytic harmonic superfield $V = q^+(\zeta, u)$ with field action:[12]

$$S_{q=1,l=1} = \int d\zeta^{(-4)} du\, (\overset{*}{\overline{q^+}}) D^{++} q^+ .$$

Likewise, in the case $q = 0, l = 2$ the second-quantized $N = 2$ superparticle represents the $N = 2$ relaxed hypermultiplet[20] described in the harmonic superspace formalism in terms of analytic harmonic superfield $V = \omega(\zeta, u)$ with field action:[12]

$$S_{q=0,l=2} = \frac{1}{2}\int d\zeta^{(-4)}\, du\, \omega(D^{++})^2\omega\,.$$

Let us stress that considering Eqs. (24.b, c) already explicitly solved, Eqs. (24.a) and (24.d) turn out to play different roles when applied to describe $N = 2$ super-Maxwell and $N = 2$ matter supermultiplets, respectively. In the former case Eq. (24.a) is the equation of motion and $D^{++}V = 0$ (Eq. (24.d)) is a gauge-fixing condition, while in the latter case $(D^{++})^l V = 0$, $l = 1, 2$, are in fact the equations of motion implying (24.a).

Recalling the interpretation of the Dirac constraint Eqs. (24) within the general scheme for covariant second quantization,[1] we could now conjecture that $N = 2$ matter- and super-Maxwell multiplets may alternatively be described in terms of general harmonic superfields $V(x, \theta^j, u)$ whose actions (which may not be local in $N = 2$ harmonic superspace):

$$S_{q,l} = \frac{1}{2}\int d(1)\,d(2)\,V(1)K_{q,l}((1),(2))\,V(2), \qquad ((1),(2) \equiv (x_{1,2}, \theta^j_{1,2}, u_{1,2})),$$

possess vast gauge invariance(s) (including the usual Yang-Mills gauge invariance as a subset) allowing the analyticity- and charge conditions (24.b, c) to be also imposed as gauge-fixing conditions. Choosing different gauge-fixing conditions for this (these) vast gauge invariance(s) one may arrive at new descriptions of the above $N = 2$ supermultiplets in terms of $N = 2$ harmonic superfields satisfying conditions different from analyticity and/or having a definite U(1) charge.

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References

1. A. Neven and P. West, *Nucl. Phys.* **B268** (1986) 125.
2. W. Siegel, *Phys. Lett.* **151B** (1985) 391, 396; A. Neveu, H. Nicolai and P. West, *Phys. Lett.* **167B** (1986) 307; E. Witten, *Nucl. Phys.* **B268** (1986) 253.
3. A. Neveu and P. West, *Phys. Lett.* **182B** (1986) 343; CERN preprint TH-4564 (1986).
4. W. Siegel, *Class. Quantum Grav.* **2** (1985) L95; *Nucl. Phys.* **B263** (1985) 93.
5. H. Terao and S. Uehara, *Z. Phys.* **C30** (1986) 647; P. Townsend, "Space-Time Supersymmetric Particles and Strings in Background Fields"—DAMPT preprint (1986); G. Sierra, *Class. Quantum Grav.* **3** (1986) L67; L. Romans, *Nucl. Phys.* **B281** (1986) 639.
6. E. S. Sokatchev, *Phys. Lett.* **169B** (1986) 209; Imperial College preprint ICTP/85-86/21.
7. M. B. Green and J. H. Schwarz, *Nucl. Phys.* **B243** (1984) 285.

8. E. R. Nissimov and S. J. Pacheva, CERN preprint TH-4630 (1986).
9. I. A. Batalin and E. S. Fradkin, *Phys. Lett.* **122B** (1983) 157; I. A. Batalin and G. A. Vilkovisky, *Phys. Rev.* **D28** (1983) 2567.
10. E. S. Fradkin and T. E. Fradkina, *Phys. Lett.* **72B** (1978) 343; M. Henneaux, *Phys. Rep.* **126** (1985) 1.
11. M. B. Green and J. H. Schwarz, *Phys. Lett.* **122B** (1983) 143; L. Brink, M. B. Green and J. H. Schwarz, *Nucl. Phys.* **B219** (1983) 437.
12. A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky and E. Sokatchev, *Class. Quantum Grav.* **1** (1984) 469; **2** (1985) 155; E. Ivanov, S. Kalitzin, N. A. Viet, and V. Ogievetsky, JINR-Dubna preprint E2-84-720 (1984).
13. A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev, *Class. Quantum Grav.* **2** (1985) 601, 617.
14. J. Wess and J. Bagger, "*Supersymmetry and Supergravity*", (Princeton Univ. Press, 1983).
15. P. A. M. Dirac, "*Lectures on Quantum Mechanics*", (Yeshiva Univ., New York, 1964).
16. S. Gates, M. Grisaru, M. Rocek and W. Siegel, "*Superspace*", (Benjamin Cummings, 1983).
17. E. S. Sokatchev and D. T. Stoyanov, *Mod. Phys. Lett. A* **1** (1986) 577; B. L. Aneva, I. H. Katzarov, D. T. Stoyanov, "*On Nonrelativistic Supersymmetry*" submitted to *Modern Physics Letter A.*
18. R. Casalbuoni, *Phys. Lett.* **62B** (1976) 49; L. Brink and J. H. Schwarz, *Phys. Lett.* **100B** (1981) 310; I. Bengtsson and M. Cederwall, Goteborg preprint 84-21 (1984).
19. P. Fayet, *Nucl. Phys.* **B113** (1976) 135; M..Sohnius, *Nucl. Phys.* **B138** (1978) 109.
20. P. Howe, K. Stelle and P. Townsend, *Nucl. Phys.* **B214** (1983) 519.